EXERCISE – III HINTS & SOLUTIONS

Sol.1 A, B, C in A.P. $\Rightarrow 2B = A + C$

$3B = \pi \Rightarrow \quad B = 60^\circ$

$$\frac{b}{c} = \frac{\sqrt{3}}{\sqrt{2}} \Rightarrow \frac{\sin B}{\sin C} = \frac{\sqrt{3}}{\sqrt{2}} \Rightarrow \frac{\sqrt{3}}{2\sin c} = \frac{\sqrt{3}}{\sqrt{2}}$$

$$\Rightarrow \sin C = \frac{1}{\sqrt{2}} \Rightarrow c = 45^\circ \ \& \ A = 75^\circ$$

Sol.2 $b\cos(A-\theta) + a\cos(B+\theta) = c\cos\theta$

L.H.S. $= b\cos A\cos\theta + b\sin A\sin\theta + a\cos B\cos\theta - a\sin B\sin\theta$

$= \cos\theta\,(a\cos B + b\cos A)$

$+ \sin\theta\,(b\sin A - a\sin B)$

$= c\cos\theta =$ R.H.S. $\left(\because \frac{a}{\sin A} = \frac{b}{\sin B}\right)$

Sol.3 $\cos A + \cos B = 4\sin^2\left(\frac{c}{2}\right)$

$$\Rightarrow 2\cos\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right) = 4\sin^2\frac{C}{2}$$

$$\Rightarrow \cos\left(\frac{A-B}{2}\right) = 2\sin\frac{C}{2}$$

$$\Rightarrow \cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$$

$$\Rightarrow 2\sin\frac{A}{2}\sin\frac{B}{2} = \sin\frac{C}{2}$$

$$\Rightarrow 2\sqrt{\frac{(s-b)(s-c)}{bc}} \times \sqrt{\frac{(s-a)(s-c)}{ac}}$$

$$= \sqrt{\frac{(s-a)(s-b)}{ab}}$$

$\Rightarrow 2(s-c) = c \Rightarrow a + b - c = c$

$\Rightarrow a + b = 2c \Rightarrow a, c, b$ in A.P.

Sol.4 $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$

$$\Rightarrow \frac{\sin(B+C)}{\sin(A+B)} = \frac{\sin(A-B)}{\sin(B-C)}$$

$\Rightarrow \sin(B+C)\sin(B-C)$

$= \sin(A+B)\sin(A-B)$

$\Rightarrow \sin^2B - \sin^2C = \sin^2A - \sin^2B$

$\Rightarrow 2\sin^2B = \sin^2C + \sin^2A$

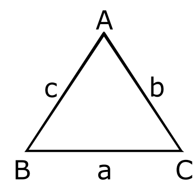
$\Rightarrow 2b^2 = a^2 + c^2 \Rightarrow a^2, b^2, c^2$ in A.P.

Sol.5 $x^3 - Px^2 + Qx - R = 0 \begin{cases} a^2 \\ b^2 \\ c^2 \end{cases}$

$\therefore a^2 + b^2 + c^2 = P$

$a^2b^2 + b^2c^2 + c^2a^2 = Q$

$a^2b^2c^2 = R$



$$\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$$

$$= \frac{bc\cos A + a(c\cos B + b\cos C)}{abc}$$

$$= \frac{\frac{1}{2}(b^2+c^2-a^2)+a^2}{abc} = \frac{a^2+b^2+c^2}{2abc} = \frac{p}{2\sqrt{R}}$$

Sol.6 a, b, c in A.P. $\Rightarrow 2b = a + c$

Now $\tan\frac{A}{2} + \tan\frac{C}{2}$

$$= \tan\left(\frac{A+C}{2}\right)\left(1-\tan\frac{A}{2}\tan\frac{C}{2}\right)$$

$$= \cot\frac{B}{2}\left(1-\frac{\Delta}{s(s-a)}.\frac{\Delta}{s(s-c)}\right)$$

$$= \cot\frac{B}{2}\left(1-\frac{s(s-a)(s-b)(s-c)}{s(s-a)s(s-c)}\right)$$

$$= \cot\frac{B}{2}\left(1-\frac{s-b}{s}\right) = \frac{b}{s}\cot\frac{B}{2}$$

$$= \frac{2b}{a+b+c}\cot\frac{B}{2} = \frac{2}{3}\cot\frac{B}{2}$$

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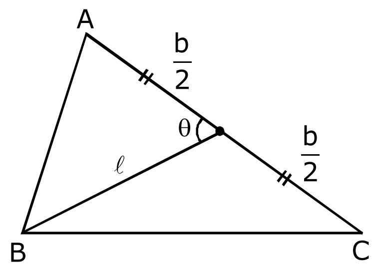
Sol.7 $\ell^2 = \frac{1}{4}(2a^2 + 2c^2 - b^2)$



$$\cos\theta = \frac{\ell^2 + \frac{b^2}{4} - c^2}{\ell b}$$

& $\frac{\Delta}{2} = \frac{1}{2} \cdot \ell\left(\frac{b}{2}\right)\sin\theta \Rightarrow \sin\theta = \frac{2\Delta}{\ell b}$

$$\frac{\sin\theta}{\cos\theta} = \frac{2\Delta}{\ell b} \times \frac{4\ell b}{4\ell^2 + b^2 - 4c^2}$$

$$\Rightarrow \tan\theta = \frac{8\Delta}{2a^2 + 2c^2 - b^2 + b^2 - 4c^2}$$

$$= \frac{8\Delta}{2(a^2 - c^2)} = \frac{4\Delta}{(a^2 - c^2)}$$

Sol.8 $a = 6,\ b = 3$ & $\cos(A - B) = \frac{4}{5}$

$$\Rightarrow \tan\left(\frac{A-B}{2}\right) = \sqrt{\frac{1 - \frac{4}{5}}{1 + \frac{4}{5}}} = \frac{1}{3}$$

$\{\therefore 0 < A - B < \frac{\pi}{3}\}$

$$\Rightarrow \tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\frac{C}{2}$$

$$\Rightarrow \frac{1}{3} = \frac{6-3}{6+3}\cot\frac{C}{2} \Rightarrow \frac{C}{2} = \frac{\pi}{4} \Rightarrow C = 90^\circ$$

area $\Delta ABC = \frac{1}{2} \cdot 6 \cdot 3 = 9$ sq. units

Sol.9 $\angle A = 30^\circ$

$$\frac{1}{2} bc\sin 30^\circ = \frac{\sqrt{3}a^2}{4} \Rightarrow bc = \sqrt{3}\,a^2$$

$$\Rightarrow \sin B \sin C = \sqrt{3}\sin^2 A = \frac{\sqrt{3}}{4}$$

$$\Rightarrow 2\sin B \sin C = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos(B - C) - \cos(B + C) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos(B - C) = \frac{\sqrt{3}}{2} + (-\cos A)$$

$$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = 0 \Rightarrow \cos(B - C) = 0$$

$\therefore B - C = 90$ ...(i) & $B + C = 150$ ...(ii)

by (i) & (ii) $\Rightarrow B = 120^\circ$ & $C = 30^\circ$

$\Rightarrow B = 4C$ or $C = 4B$

Sol.10 $x^3 - x^2(4R + r) + xs^2 - rs^2 = 0$ (roots r_1, r_2, r_3)

$$r_1 + r_2 + r_3 = \underbrace{\frac{\Delta}{s-a} + \frac{\Delta}{s-b}} + \underbrace{\frac{\Delta}{s-c} - \frac{\Delta}{s}} + \frac{\Delta}{s}$$

$$= \frac{\Delta c}{(s-a)(s-b)} + \frac{\Delta c}{s(s-c)} + \frac{\Delta}{s}$$

$$= \Delta c\left[\frac{s^2 - sc + s^2 - (a+b)s + ab}{s(s-a)(s-b)(s-c)}\right] + r$$

$$= \frac{\Delta c}{\Delta^2}[2s^2 - s(a + b + c) + ab] + r$$

$$= \frac{abc}{\Delta} + r = 4R + r$$

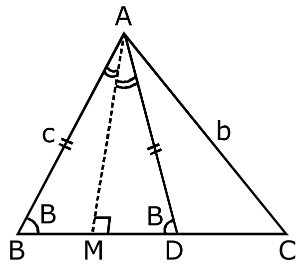
$$r_1r_2 + r_2r_3 + r_3r_1 = s^2\tan\frac{A}{2}\tan\frac{B}{2} + s^2\tan\frac{B}{2}\tan\frac{C}{2} + s^2\tan\frac{C}{2}\tan\frac{A}{2}$$

$= s^2(1) = s^2$

$$r_1 \cdot r_2 \cdot r_3 = \frac{\Delta^3 s}{(s-a)(s-b)(s-c)s} = \Delta s$$

$$= (rs)\, s = rs^2$$

Sol.11 BD = DC = $\frac{a}{2}$



$$\cos B = \frac{c^2 + \frac{a^2}{4} - c^2}{ac}$$

$$\Rightarrow \cos B = \frac{a}{4c} \ \&$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \Rightarrow \frac{a^2 + c^2 - b^2}{2ac} = \frac{a}{4c}$$

$$\Rightarrow 2c^2 + 2a^2 - 2b^2 = a^2 \Rightarrow a^2 = 2(b^2 - c^2)$$

Sol.12 $a \tan A + b \tan B = (a + b) \tan\left(\frac{A+B}{2}\right)$

$$\Rightarrow \frac{\sin^2 A}{\cos A} + \frac{\sin^2 B}{\cos B} = (\sin A + \sin B) \tan\left(\frac{A+B}{2}\right)$$

$$\Rightarrow \frac{1}{\cos A} - \cos A + \frac{1}{\cos B} - \cos B$$

$$= \frac{2\sin^2\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)}$$

$$\Rightarrow \frac{(\cos A + \cos B)}{\cos A \cos B} - (\cos A + \cos B)$$

$$= \frac{2\sin^2\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)}$$

$$\Rightarrow (\cos A + \cos B)\left(\frac{1}{\cos A \cos B} - 1\right)$$

$$= \frac{2\sin^2\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)}$$

$$\Rightarrow 2\cos^2\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\left(\frac{1}{\cos A \cos B} - 1\right)$$

$$= 2\sin^2\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\left\{\cos\frac{A-B}{2} \neq 0 \Rightarrow \cos^2\frac{A+B}{2} \neq \frac{\pi}{2} \Rightarrow A - B \neq \pi\right\}$$

$$\Rightarrow \frac{\cos^2\left(\frac{A+B}{2}\right)}{\cos A \cos B} = \cos^2\left(\frac{A+B}{2}\right) + \sin^2\left(\frac{A+B}{2}\right) = 1$$

$$\Rightarrow 2\cos^2\left(\frac{A+B}{2}\right)$$

$$= 2\cos A \cos B = \cos(A + B) + \cos(A - B)$$

$$\Rightarrow 2\cos^2\left(\frac{A+B}{2}\right) = 2\cos^2\left(\frac{A+B}{2}\right) - 1 + \cos(A - B)$$

$$\Rightarrow \cos(A - B) = 1 \Rightarrow A - B = 0 \Rightarrow A = B$$

ΔABC is isosceles Δ

Aliter :

$$a\tan A + b\tan B = a\tan\left(\frac{A+B}{2}\right) + b\tan\left(\frac{A+B}{2}\right)$$

$$\Rightarrow a\left(\tan A - \tan\left(\frac{A+B}{2}\right)\right) + b\left(\tan B - \tan\left(\frac{A+B}{2}\right)\right) = 0$$

$$\Rightarrow \frac{a\sin\left(A - \frac{A+B}{2}\right)}{\cos A\cos\left(\frac{A+B}{2}\right)} + \frac{b\sin\left(B - \frac{A+B}{2}\right)}{\cos B\cos\left(\frac{A+B}{2}\right)} = 0$$

$$\Rightarrow \frac{a\sin\left(\frac{A-B}{2}\right)}{\cos A\cos\left(\frac{A+B}{2}\right)} - \frac{b\sin\left(\frac{A-B}{2}\right)}{\cos B\cos\left(\frac{A+B}{2}\right)} = 0$$

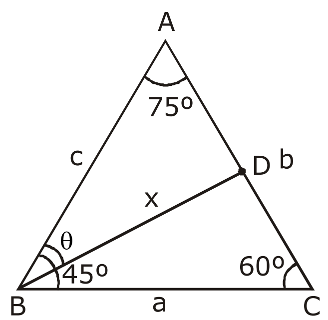
$$\Rightarrow \frac{\sin\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)}\left[\frac{a}{\cos A} - \frac{b}{\cos B}\right] = 0$$

$$\Rightarrow \frac{\sin\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)} \frac{\sin(A-B)}{\cos A \cos B} = 0$$

$$\Rightarrow \frac{A-B}{2} = 0 \text{ or } A - B = 0 \Rightarrow A = B$$

$\therefore \Delta ABC$ is isosecles Δ

Sol.13 $\angle C = 60^\circ$, $\angle A = 75^\circ$, $\Delta_{BAD} = \sqrt{3}\ \Delta_{BCD}$



$$\Rightarrow \frac{1}{2} \text{ cx } \sin\theta = \sqrt{3} \cdot \frac{1}{2} \cdot \text{ax } \sin(45^\circ - \theta)$$

$$\Rightarrow \frac{\sin(45-\theta)}{\sin\theta} = \frac{c}{\sqrt{3}a} = \frac{\sin C}{\sqrt{3}\sin A} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}$$

$$\Rightarrow \frac{\sin(45^\circ-\theta)}{\sin\theta} = \frac{\sqrt{2}}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{\sqrt{2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \cot\theta - \frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{\sqrt{2}}$$

$$\Rightarrow \cot\theta = \sqrt{3} \Rightarrow \theta = 30^\circ$$

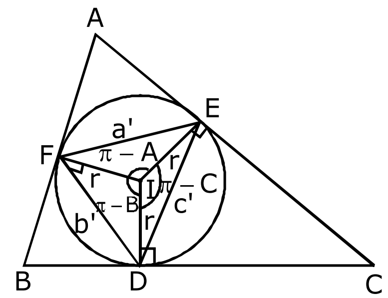
Sol.14 $$\frac{\pi r^2}{\Delta} = \frac{\pi(4R)^2 \sin^2\frac{A}{2}\sin^2\frac{B}{2}\sin^2\frac{C}{2}}{abc/4R}$$

$$\frac{\pi(4R)^3 \sin^2\frac{A}{2}\sin^2\frac{B}{2}\sin^2\frac{C}{2}}{(2R\sin A)(2R\sin B)(2R\sin C)}$$

$$= \frac{\pi(4R)^3 \sin^2\frac{A}{2}\sin^2\frac{B}{2}\sin^2\frac{C}{2}}{2^3R^3\left(2\sin\frac{A}{2}\cos\frac{A}{2}\right)\left(2\sin\frac{B}{2}\cos\frac{B}{2}\right)\left(2\sin\frac{C}{2}\cos\frac{C}{2}\right)}$$

$$= \pi\,\Pi\left(\tan\frac{A}{2}\right) = \frac{\pi}{\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}}$$

Sol.15 (i) sine law in ΔDEF



$$\frac{a'}{\sin D} = \frac{b'}{\sin E} = \frac{c'}{\sin F} = 2r$$

$$\Rightarrow \frac{a'}{\sin\left(\frac{\pi}{2}-\frac{A}{2}\right)} = \frac{b'}{\sin\left(\frac{\pi}{2}-\frac{B}{2}\right)} = \frac{c'}{\sin\left(\frac{\pi}{2}-\frac{C}{2}\right)} = 2r$$

$$\Rightarrow a = 2r\cos\frac{A}{2},\ b' = 2r\cos\frac{B}{2},\ c' = 2r\cos\frac{C}{2}$$

(ii) $\angle EIF = \pi - A$

$\angle EDF = \frac{\pi - A}{2}$ angle of ΔDEF

are $\frac{\pi}{2} - \frac{A}{2}$, $\frac{\pi}{2} - \frac{B}{2}$, $\frac{\pi}{2} - \frac{C}{2}$

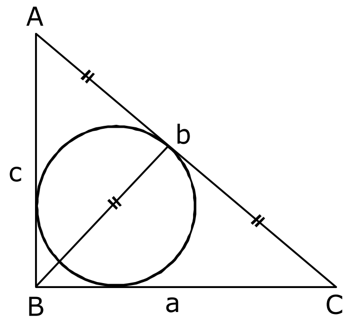
(iii) $$\Delta_{DEF} = \frac{1}{2}\left(2r\cos\frac{A}{2}\right)\left(2r\cos\frac{B}{2}\right)\sin\left(\frac{\pi}{2}-\frac{C}{2}\right)$$

$$= 2r^2 \cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

$$= 2r^2\sqrt{\frac{s(s-a)}{bc}} \cdot \sqrt{\frac{s(s-b)}{ac}} \times \sqrt{\frac{s(s-c)}{ab}}$$

$$= \frac{2r^2 s\Delta}{abc} \times \frac{s}{s} = \frac{2(rs)^2\Delta}{abc.s} = \frac{2\Delta^3}{abc.s}$$

Sol.16 If circumcentre lies on incircle



$\Rightarrow$ Δ is isoscels right angled triangle

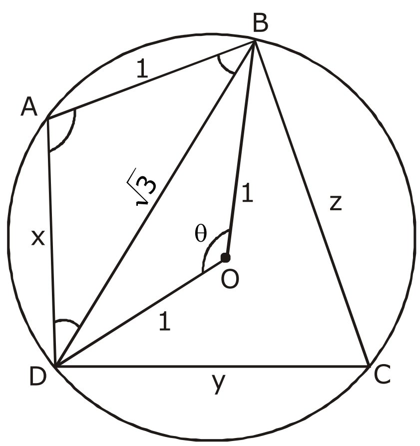
Let B = 90º & A = C

$\Rightarrow$ 2A = 90º $\Rightarrow$ A = 45º, C = 45º

cosA + cos B + cos C = 2 cos 45º

$= 2.\dfrac{1}{\sqrt{2}} = \sqrt{2}$

Sol.17 Area ABCD = $\dfrac{3\sqrt{3}}{4}$,



Let AD = x

In ΔOBD, $\cos\theta = \dfrac{1^2 + 1^2 - 3}{2} = -\dfrac{1}{2}$

$\Rightarrow$ θ = 120º $\Rightarrow$ $\angle$C = 60º $\Rightarrow$ $\angle$A = 120º

In ΔABD, $\cos A = \dfrac{x^2 + 1 - 3}{2x} \Rightarrow -\dfrac{1}{2} = \dfrac{x^2 - 2}{2x}$

$\Rightarrow$ $x^2 + x - 2 = 0$ $\Rightarrow$ $(x + 2)(x - 1) = 0$

$\Rightarrow$ x = 1 $\{\therefore x \neq -2\}$

$\Delta_{ABD} = \dfrac{1}{2}$. 1 1. sin 120º $= \dfrac{\sqrt{3}}{4}$

$\Rightarrow$ $\Delta_{BDC} = \dfrac{3\sqrt{3}}{4} - \dfrac{\sqrt{3}}{4} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \Delta_{BDC} = \dfrac{\sqrt{3}}{2} = \dfrac{1}{2}$. y. z. sin 60º $\Rightarrow$ yz = 2

In ΔBDC, $\cos 60º = \dfrac{y^2 + z^2 - 3}{2yz} = \dfrac{1}{2}$

$\Rightarrow$ $y^2 + \left(\dfrac{2}{y}\right)^2 - 3 = 2$

$\Rightarrow$ $y^4 - 3y^2 - 2y^2 + 4 = 0$

$\Rightarrow$ $y^4 - 5y^2 + 4 = 0$

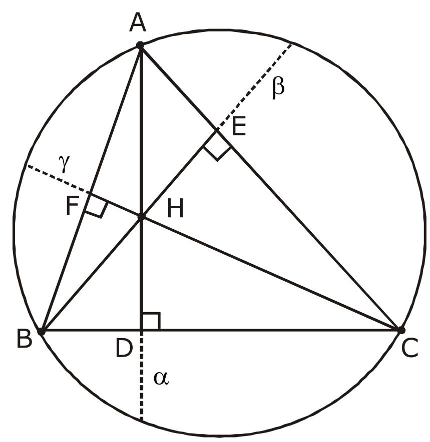
$\Rightarrow$ $(y^2 - 1)(y^2 - 4) = 0$

$\Rightarrow$ y = 1 or y = 2 $\{\therefore y, z > 0\}$

If y = 1 $\Rightarrow$ z = 2, If y = 2 $\Rightarrow$ z = 1

Three sides are 1, 1, 2 or 1, 2, 1

Sol.18 HD = α = 2R cos B cos C



HE = β = 2R cosA cos C

HF = γ = 2R cosA cos B

L.H.S. $= \dfrac{a}{\alpha} + \dfrac{b}{\beta} + \dfrac{c}{\gamma}$

$$= \frac{2R\sin A}{2R\cos B\cos C} + \frac{2R\sin B}{2R\cos A\cos C} + \frac{2R\sin C}{2R\cos A\cos B}$$

$$= \frac{\sin 2A + \sin 2B + \sin 2C}{2\cos A\cos B\cos C} = \frac{4\sin A\sin B\sin C}{2\cos A\cos B\cos C}$$

= 2 tanA tanB tanC= 2 (tan A + tan B + tan C)

Sol.19 sinA sinB sinC = p, cosA cosB cosC = q

$\Rightarrow$ tanA . tan B tan C = $\frac{p}{q}$ ...(i)

$\Rightarrow$ tanA + tanB + tanC = $\frac{p}{q}$ ...(ii)

tanA tan B + tanB tanC + tanC tanA

$$= \frac{\sin B(\sin A\cos C + \cos A\sin C) + \sin A\sin C\cos B}{\cos A\cos B\cos C}$$

$$= \frac{\sin^2 B + \sin A\sin C\cos B}{q}$$

$$= \frac{1}{q}\,[1 - \cos B\,(\cos B - \sin A\sin C)]$$

$$= \frac{1}{q}\,[1 - \cos B\{- \cos (A + C) - \sin A \sin C\}]$$

$$= \frac{1}{q}\,[1 - \cos B\,(- \cos A \cos C)]$$

$$= \frac{1}{q}\,[1 + q] \Rightarrow \Sigma \tan A \tan B = \frac{(1+q)}{q} \quad ...(iii)$$

from (i), (ii), & (iii)

If tanA, tanB, tan C are of equation is

$$x^3 - \frac{p}{q}x^2 + \left(\frac{1+q}{q}\right)x - \frac{p}{q} = 0$$

$\Rightarrow$ $qx^3 - px^2 + (1 + q)x - p = 0$

Sol.20 L.H.S. = $\frac{b-c}{r_1} + \frac{c-a}{r_2} + \frac{a-b}{r_3}$

$$= \frac{1}{\Delta}[(b - c)(s - a) + (c - a)(s - b) + (a - b)(s - c)]$$

$$= \frac{1}{\Delta}[\{(s - c) - (s - b)\} . (s - a) + \{(s - a) - (s - c)\}(s - b) + \{(s - b) - (s - a)\}(s - c)]$$

$$= \frac{1}{\Delta}(0) = 0$$

Sol.21 L.H.S. = a cotA + b cotB + c cot C

= 2R cosA + 2R cosB + 2R cos C

= 2R [cosA + cosB + cosC]

$$= 2R\left[1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right]$$

$$= 2R + 2\left(4R\,\pi\sin\frac{A}{2}\right) = 2R + 2r = 2(R + r)$$

Sol.22 $\frac{r_1}{(s-b)(s-c)} + \frac{r_2}{(s-c)(s-a)} + \frac{r_3}{(s-a)(s-b)}$

$$= \frac{3\Delta}{(s-a)(s-b)(s-c)} \times \frac{s}{s} = \frac{3\Delta s}{\Delta^2} = \frac{3s}{\Delta} = \frac{3}{r}$$

Sol.23

$$\frac{r_1 - r}{a} + \frac{r_2 - r}{b} = \frac{1}{a}\left[\frac{\Delta}{s-a} - \frac{\Delta}{s}\right] + \frac{1}{b}\left[\frac{\Delta}{s-b} - \frac{\Delta}{s}\right]$$

$$\text{L.H.S.} = \frac{\Delta}{s(s-a)} + \frac{\Delta}{s(s-b)} = \frac{\Delta}{s}\left[\frac{s-a+s-b}{(s-a)(s-b)}\right]$$

$$= \frac{\Delta(2s-a-b)}{s(s-a)(s-b)} = \frac{\Delta c}{s(s-a)(s-b)} = \frac{\Delta c}{\left(\frac{\Delta^2}{(s-c)}\right)}$$

$$= \frac{c}{\left(\frac{\Delta}{s-c}\right)} = \frac{c}{r_3}$$

Sol.24 $\frac{abc}{s}\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

$$= \frac{abc}{s}\sqrt{\frac{s(s-a)}{bc}}\sqrt{\frac{s(s-b)}{ac}}\sqrt{\frac{s(s-c)}{ab}}$$

$$= \frac{s}{s}\Delta = \Delta$$

Sol.25. $(r_1+r_2)\tan\frac{C}{2} = \Delta\left(\frac{1}{(s-a)} + \frac{1}{(s-b)}\right)\frac{\Delta}{s(s-c)}$

$$= \frac{\Delta^2 c}{s(s-a)(s-b)(s-c)} = c$$

Now $(r_3 - r)\cot\frac{C}{2} = \Delta\left(\frac{1}{s-c} - \frac{1}{s}\right)\frac{s(s-c)}{\Delta}$

$$= \frac{c}{s(s-c)} \times s(s-c) = c$$

$$\Rightarrow (r_1 + r_2)\tan\frac{C}{2} = (r_3 - r)\cot\frac{C}{2} = c$$